Econometrics I
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Exercise sheet 1

1. Show that for the OLS estimate the fitted values and residuals are invariant to nonsingular linear transformations of the independent variables.

2. Show that the OLS estimates $\hat{\beta}_1$ and $\hat{\beta}_2$ from the linear regression

$$y_i = \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2_i} + e_i$$

are identical to those from

$$(y_i - \overline{y}) = \beta_1(x_{1i} - \overline{x}_1) + \beta_2(x_{2i} - \overline{x}_{2_i}) + e_i,$$

where $\overline{z}$ refers to the sample mean of z_i.

3. Based on n observations on a variable y and k independent variables we obtained the OLS estimator $\hat{\beta}_n$. Assume that a new observation (observation $n+1$) is available for all variables. Under which condition does the new estimator $\hat{\beta}_{n+1}$ differ from $\hat{\beta}_n$?

4. Assume that the true model is

$$y = X_1\beta_1 + X_2\beta_2 + e,$$

where the usual conditions on e hold. The econometrician, however, estimates

$$y = X_1\gamma_1 + \tilde{e}.$$

Under which conditions is $\hat{\beta}_1 = \hat{\gamma_1}$?

5. Consider the linear homoskedastic model with two variables,

$$y_i = x_{1i}\beta_1 + x_{2i}\beta_2 + e_i,$$

and define

$$\frac{1}{n}X'X = \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}.$$

Examine and interpret the effect of ρ on the precision of the OLS estimate of $\beta = (\beta_1 \quad \beta_2)'$.