# Evaluation of

# Vector Error Correction Models

in comparison with

**Simkins: Forecasting with Vector Autoregressive (VAR) Models Subject to Business Cycle Restrictions**

Stefan Zeugner

9851051

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Stefan Zeugner

Student at *Vienna University of Economics and B.A.*
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e-mail: h9851051@wu.edu

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## Abstract:

This paper is based on the cited article of Scott Simkins (1995): In order of producing macroeconomic forecasts, he constructed a 5-varibale VAR restricted by common characteristics of business cycles in a Monte Carlo procedure. Simkins then evaluated its performance against an unrestricted VAR and a Bayesian VAR and concluded that his procedure was only marginally superior to an unrestricted VAR and that a BVAR analysis performed much better in predicting GNP, unemployment and inflation.

In this paper I will show that a slight improvement in the specification of the unrestricted VAR and, even more, Vector Error Correction models produce forecasts able to compete with the BVAR mentioned above.

## Introduction

Originally, I was supposed to reproduce the basis of this paper, the article "Forecasting with Vector Autoregressive (VAR) Models Subject to Business Cycle Restrictions".

The author, Prof. Simkins, was primarily interested in an application of business cycle theory to forecasting of five US quarterly macroeconomic time series: Real GNP, GNP deflator, unemployment rate, real fixed investment and money supply (M1) from 1948 to 1990.

By applying a turning point procedure (Bry and Boschan), he then distinguished seven completed business cycles in the data. These cycles were normalized by their mean and divided into nine stages – the first trough (start), the peak and the second trough (end) and three successive thirds for the expansion and the contraction phase – and finally the mean of these stages (plus/minus standard deviation bounds) was computed for each of the five variables.

Then Simkins estimated an ordinary, 6-lag level VAR and applied multinormal drawing procedures to its parameters and errors in order to conduct a Monte Carlo simulation for the whole sample period. Then the same turning point and stage procedure as above was applied to the simulation outcomes and only those corresponding to the "historical" business cycle patterns (for each variable – this resulted in a selection of about 10% of the simulated paths) were selected as good enough for conducting dynamic forecasts (1 to 8 steps ahead) for three arbitrarily chosen periods (1987:3-1989:2, 1988:2-1990:1, 1989:1-1990:4). The author then evaluated the "fit" of these predictions by Theil's U-Statistic of GNP, Deflator and unemployment rate forecasts.

Besides his own restricted VAR, Simkins did an evaluation of well-known, more established VAR techniques: an unrestricted, normal VAR and a Bayesian VAR (BVAR) with Minnesota Priors (this is a technique of imposing prior distributions near to random walk to the VAR parameters and obtaining the "best" distribution by successive re-estimation of the VAR's final distribution by Monte Carlo Methods).

As I mentioned above, I was supposed to reproduce the paper and I invested a lot of energy in understanding the theory of Bayesian and Monte Carlo techniques in order to calculate the restricted VAR model and the BVAR. But the means I had were inadequate: no courses and experts regarding the matter at university, and the software Eviews, Mathematica, MS Excel and VBA. After three days I concluded that accomplishing my task would be a matter of weeks, rather than days. Therefore I chose a different path:

Simkins wrote his paper in 1994, when Vector Error Correction (VEC) models did already exist: Nevertheless he did not consider them for evaluating the performance of his own model or for applying his procedure to them. In the following pages, I will show how I estimated VEC models that even beat the Theil U Statistic for BVARs. Moreover I will demonstrate that a slight change in the specification of the VAR (not to levels, but to their logarithms) improves its performance considerably. These types of models are much more simple to estimate than a BVAR or Simkins' variant (and they could provide a better basis for applying Simkins' or Bayesian procedures).

I will first introduce Simkins' unrestricted VARs and consider some improvements. Then I will evaluate certain variants of VECs and their different performance regarding different questions.

## Simkins' unrestricted VAR

Simkins estimated a simple 6-lag VAR with constants, corresponding to a macroeconomic model by Litterman. Figure 1 shows its estimation output.

| | Deflator | Investment | M1 | GNP | unemployment |
|---|---|---|---|---|---|
| R-squared | 0.999950 | 0.999396 | 0.999834 | 0.996594 | 0.975708 |
| Adj. R-squared | 0.999937 | 0.999247 | 0.999793 | 0.995750 | 0.969685 |
| Sum sq. resids | 6.444210 | 53068.20 | 610.8618 | 9375.864 | 10.92836 |
| S.E. equation | 0.230777 | 20.94230 | 2.246874 | 8.802640 | 0.300528 |
| F-statistic | 80186.42 | 6677.206 | 24289.37 | 1180.301 | 161.9992 |
| Log likelihood | 24.53443 | -660.6930 | -321.3939 | -528.9517 | -15.60716 |
| Akaike AIC | 0.085073 | 9.101224 | 4.636762 | 7.367785 | 0.613252 |
| Schwarz SC | 0.701786 | 9.717937 | 5.253474 | 7.984498 | 1.229965 |
| Mean dependent | 51.99013 | 2337.464 | 256.1728 | 375.7796 | 5.724013 |
| S.D. dependent | 29.12900 | 763.0011 | 156.0972 | 135.0275 | 1.726051 |
| Determinant Residual Covariance | | 102.4292 | | | |
| Log Likelihood | | -1430.210 | | | |
| Akaike Information Criteria | | 20.85803 | | | |
| Schwarz Criteria | | 23.94159 | | | |

***Figure 1: Estimation output of Simins' VAR***

The model was evaluated versus the two others by Theil's U Statistic: This measure divides the root mean squared error (RMSE) of the models forecast by the RMSE of the naïve forecast: the naïve forecast is simply taking the last value in the sample (the forecast's starting point) as a prediction for the time series. The more the value of Theil's U Statistic is close to zero, the better is the fit of the underlying forecast. A value below 1 indicates that the model performs better than the naive forecast.

This measure is applied to Real GNP, GNP Deflator and unemployment rate forecasts – the performance of Simkins' three models is shown in Figure 2.

| Variable | Steps ahead (k) | Unrestricted VAR model | Restricted VAR model | Bayesian VAR model |
|---|---|---|---|---|
| Real GNP | 1 | 1.062 | 1.043 | 0.303 |
| | 2 | 1.227 | 1.186 | 0.298 |
| | 3 | 1.198 | 1.132 | 0.311 |
| | 4 | 1.158 | 1.060 | 0.366 |
| | 5 | 1.124 | 0.993 | 0.445 |
| | 6 | 1.116 | 0.977 | 0.549 |
| | 7 | 1.135 | 1.007 | 0.648 |
| | 8 | 1.157 | 1.039 | 0.794 |
| GNP Deflator | 1 | 0.551 | 0.510 | 0.290 |
| | 2 | 0.651 | 0.582 | 0.289 |
| | 3 | 0.721 | 0.605 | 0.284 |
| | 4 | 0.786 | 0.621 | 0.274 |
| | 5 | 0.838 | 0.634 | 0.262 |
| | 6 | 0.869 | 0.631 | 0.253 |
| | 7 | 0.900 | 0.634 | 0.252 |
| | 8 | 0.941 | 0.646 | 0.266 |
| Unemployment Rate | 1 | 3.152 | 3.117 | 0.656 |
| | 2 | 4.332 | 4.212 | 0.635 |
| | 3 | 5.601 | 5.342 | 0.779 |
| | 4 | 6.569 | 6.124 | 0.939 |
| | 5 | 7.861 | 7.160 | 1.302 |
| | 6 | 8.227 | 7.406 | 1.523 |
| | 7 | 8.606 | 7.750 | 1.945 |
| | 8 | 8.529 | 7.775 | 2.458 |

***Figure 2: Theil U statistics of Simkins' three models' 1-8-step-ahead***

The Theil U Statistics for the unrestricted VAR were computed by seven 1 to 8-step ahead forecasts in the period 1987:3 to 1990:4. Thus the starting periods for the seven forecasts are from 1987:2 to 1988:4. It can easily be seen that the dynamic forecasts become the less accurate, the more they are ahead of their starting period. Simkin's methods are only marginally superior to the predictions by an unrestricted VAR, whereas the Bayesian VAR performs much better than the simple and the "theoretical" VAR.

## A VAR in logs

However, some problems were not considered in this approach: First, a VAR is a linear model, i.e. it does not capture non-linear elements, elements existing certainly in level series of GNP, deflator, money supply and investment (especially concerning their exponential growth). The easiest way to respond to this problem is to linearize the data by taking the logs of the levels.

Second, the lag length of 6 is not the optimal choice, if one considers selection criteria based on log likelihood. A log-estimation of different lag lengths and choice by the Schwarz criterion (SC) and the Akaike info criterion (AIC) lead to the conclusion that a VAR with lag three would be optimal: SC and AIC are the highest for a 2-lag VAR[1], plus one lag for being sure to capture additional information (for the case that the minimum lies between lag two and lag three). One might consider a lag selection by an LR statistic, too: This is a measure of testing the null-hypothesis of adding parameters to the model does not change it significantly towards the "good" direction. Given the high number of added restrictions (parameters of the equations) per lag (25, the number of degrees of freedom in a $Chi^2$-distribution per adding one lag) the resulting p-values prefer the 3-lag model to every higher-lag model[2].

| | Deflator | Investment | M1 | GNP | Unemployment |
|---|---|---|---|---|---|
| R-squared | 0.999911 | 0.997028 | 0.999874 | 0.999419 | 0.967683 |
| Adj. R-squared | 0.999902 | 0.996721 | 0.999861 | 0.999359 | 0.964340 |
| Sum sq. resides | 0.004090 | 0.072261 | 0.006796 | 0.012190 | 0.476889 |
| S.E. equation | 0.005311 | 0.022324 | 0.006846 | 0.009169 | 0.057349 |
| F-statistic | 109195.5 | 3243.435 | 76727.68 | 16637.24 | 289.4563 |
| Log likelihood | 623.2854 | 392.1148 | 582.4173 | 535.3801 | 240.2119 |
| Akaike AIC | -7.543918 | -4.672234 | -7.036240 | -6.451926 | -2.785241 |
| Schwarz SC | -7.237691 | -4.366007 | -6.730013 | -6.145700 | -2.479014 |
| Mean dependent | 3.837678 | 5.875014 | 5.425485 | 7.710055 | 1.696497 |
| S.D. dependent | 0.537401 | 0.389856 | 0.580659 | 0.362215 | 0.303693 |
| Determinant Residual Covariance | | 4.22E-20 | | | |
| Log Likelihood | | 2448.957 | | | |
| Akaike Information Criteria | | -29.42804 | | | |
| Schwarz Criteria | | -27.89691 | | | |

***Figure 3: Estimation output for a VAR in logs (3 lags)***

But Simkins wanted to conduct dynamic forecasts up to eight periods ahead. Considering this aim, more lags would certainly add a bit more "real" information into far-ahead forecasts, even if their short-term performance would suffer. Concerning that, a 6-lag (and maybe an 8-lag) VAR seem to be the best choice because there log-likelihoods increase considerably over lag 5 and 7. Nevertheless, for the sake of a short paper I will only analyze the three lag VAR: The estimation output of such a VAR is shown in Figure 3.

[1] The AIC is even higher with higher lag number (6 and 8), but very slightly.

[2] The p-values are computed by the following procedure: one minus the Chi-squared distribution of two times the log-likelihood of the lower-lag VAR minus log-likelihood of the higher-lag VAR, with the as many degrees of freedom as the difference of parameters between the two. The resulting p-values are 0.99, 0.567 and 0.687 for the four-, six, and eight-lag VARs versus the three-lag VAR, respectively.

As seen in the Theil U table in the appendix[3], this log-VAR provides a much better fit then the original one, a feature that also marks the covariance matrix of the residuals: A comparison of the two determinants of each's residual covariance matrix shows a value >100 for the original VAR and a value near to zero for the log-VAR. Since a linearly dependent covariance matrix seems unlikely, the zero-value must be due to very small covariances – but these are caused by the transformation into log-units, and must not be due to a real improvement of the model. The same goes for the "criterions": The lower AIC and SC values of the log-VAR can not be considered as an improvement since the dependent variable has changed.

Covariance matrix of the VAR in levels

| | Deflator | GNP | Investment | M1 | Unempl. |
|---|---|---|---|---|---|
| Deflator | 0.046707 | 0.321017 | -0.155511 | -0.045697 | -0.007788 |
| GNP | 0.321017 | 62.17747 | 87.38736 | 3.884951 | -0.859825 |
| Investment | -0.155511 | 87.38736 | 362.9078 | 4.918358 | -2.739014 |
| M1 | -0.045697 | 3.884951 | 4.918358 | 5.019088 | -0.107527 |
| Unempl. | -0.007788 | -0.859825 | -2.739014 | -0.107527 | 0.073740 |

Covariance matrix of the VAR in logs

| | Deflator | GNP | Investment | M1 | Unempl. |
|---|---|---|---|---|---|
| Deflator | 2.54E-05 | 1.25E-05 | 6.94E-07 | -6.41E-07 | -4.84E-05 |
| GNP | 1.25E-05 | 0.000449 | 4.32E-05 | 9.43E-05 | -0.000508 |
| Investment | 6.94E-07 | 4.32E-05 | 4.22E-05 | 1.37E-05 | -7.15E-05 |
| M1 | -6.41E-07 | 9.43E-05 | 1.37E-05 | 7.57E-05 | -0.000276 |
| Unempl. | -4.84E-05 | -0.000508 | -7.15E-05 | -0.000276 | 0.002962 |

***Figure 4: Covariance matrices for Simkins' VAR and the log-VAR***

**VEC models**

As seen by a closer look at Figure 1 and 3, the high $R^2$s of the VAR models in (log-linearized) levels hint at a spurious regression problem. This does not mean that there is no relationship between our five variables, but part of the $R^2$ might only be due to the correlation of integrated data. A unit-root test on the five variables confirms this suspicion: not even the coefficient[4] of the unemployment rate is negative enough to reject the null hypothesis of an integrated time series! The residuals of the level-VAR are also integrated, whereas the residuals of the log-VAR are not.

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[3] The Theil U values for the unrestricted VAR in the appendix differ slightly from its Theil U values in Figure 2. This is due to the fact that Theil U statistics in the appendix are from the VAR I reproduced relying on Simkins' paper (and computed in MS Excel), and Figure 1 is copied from Simkins. The difference may be attributed to MS Excel's minor accuracy concerning matrix operations.

[4] The coefficient of a LS regression of the first difference of the unemployment rate versus its value lagged by one period (plus 4 lagged first differences). The unit-root test carried out was an Augmented-Dickey-Fuller Test.

The first response to this problem would be to estimate a VAR in the first differences (resp. The returns) of our five variables. Nevertheless, some important information may also be contained in the levels of the data: e.g. the so-called “productivity slowdown” in US after-war growth rates (the higher the GNP, the less its growth). Regarding that, a Vector Error Correction (VEC) Model would be the right response, principally a VAR in first differences but with correction restrictions based on the cointegration concept.

Series: LOG(DEFL) LOG(GNP) LOG(INV) LOG(MONE) LOG(UNEMP)
Lags interval: 1 to 6

| Data Trend: Rank or No. of CEs | None No Intercept No Trend | None Intercept No Trend | Linear Intercept No Trend | Linear Intercept Trend | Quadratic Intercept Trend |
|---|---|---|---|---|---|
| Akaike Information Criteria by Model and Rank | | | | | |
| 0 | -29.24852 | -29.24852 | -29.34955 | -29.34955 | -29.34029 |
| 1 | -29.34006 | -29.46143 | -29.55635 | -29.55415 | -29.55694 |
| 2 | -29.38920 | -29.50464 | -29.59161 | -29.62532 | -29.61359 |
| 3 | -29.42351 | -29.52707 | -29.58078 | -29.61583 | -29.59574 |
| 4 | -29.40625 | -29.49769 | -29.48565 | -29.55768 | -29.54971 |
| 5 | -29.28650 | -29.38928 | -29.38928 | -29.44938 | -29.44938 |
| Schwarz Criteria by Model and Rank | | | | | |
| 0 | -26.42494 | -26.42494 | -26.43184 | -26.43184 | -26.32847 |
| 1 | -26.32823 | -26.43078 | -26.45041 | -26.42938 | -26.35687 |
| 2 | -26.18913 | -26.26693 | -26.29743 | -26.29349 | -26.22529 |
| 3 | -26.03521 | -26.08229 | -26.09836 | -26.07694 | -26.01920 |
| 4 | -25.82971 | -25.84585 | -25.81499 | -25.81172 | -25.78493 |
| 5 | -25.52172 | -25.53038 | -25.53038 | -25.49635 | -25.49635 |
| L.R. Test: | Rank = 4 | Rank = 4 | Rank = 2 | Rank = 2 | Rank = 2 |

***Figure 5: Johansen Cointegration test summarizing five assumptions***

In order to know if a VEC is appropriate, a cointegration test has to be conducted. Figure 5 summarizes such a test for the number of cointegration relations, and the columns correspond to the five different assumptions concerning the structure of the VEC equations (the number of lags does not change the outcome significantly). According to its output, assumption 4[5] is selected because it sounds reasonable that imbalances in our four integrated (without the unemployment rate) variables may grow or fall with respect to time. In addition, a number of two cointegration equations may be more realistic regarding the character of our time series and the SC and AIC of assumption four seem more convincing.

[5] The cointegration equation contains constants and a linear trend.

Therefore a VEC with two cointegration equations under assumption four is estimated, one for 3 lags and one for 6 lags. Figure 7 shows their estimation outputs. The two cointegration equations yield the same output regardless of which variables are included in each of them, since they can be transformed linearly. An short look at the two lower tables shows that almost all of the variables depend significantly on at

| | *3-lag-VEC* Standard errors & t-statistics in parentheses | | *6-lag-VEC* Standard errors & t-statistics in parentheses | |
|---|---|---|---|---|
| Cointegrating Eq: | CointEq1 | CointEq2 | CointEq1 | CointEq2 |
| LOG(DEFLATOR(-1)) | 1.000000 | 0.000000 | 1.000000 | 0.000000 |
| LOG(INVESTMENT(-1)) | 0.000000 | 1.000000 | 0.000000 | 1.000000 |
| LOG(M_ONE(-1)) | -0.210937<br>(0.29787)<br>(-0.70815) | -0.575160<br>(0.23871)<br>(-2.40943) | -0.466773<br>(0.14714)<br>(-3.17220) | -0.062476<br>(0.08027)<br>(-0.77833) |
| LOG(REALGNP(-1)) | 6.397353<br>(2.88127)<br>(2.22033) | -6.304075<br>(2.30905)<br>(-2.73016) | 3.968452<br>(1.20802)<br>(3.28508) | -0.932555<br>(0.65899)<br>(-1.41513) |
| LOG(UNEMPL(-1)) | 0.648705<br>(0.40834)<br>(1.58864) | -0.789048<br>(0.32724)<br>(-2.41119) | 0.323661<br>(0.16503)<br>(1.96119) | -0.028291<br>(0.09003)<br>(-0.31425) |
| @TREND(48:1) | -0.057139<br>(0.02594)<br>(-2.20279) | 0.048589<br>(0.02079)<br>(2.33740) | -0.035204<br>(0.01102)<br>(-3.19345) | -1.13E-05<br>(0.00601)<br>(-0.00187) |
| C | -48.31831 | 43.10782 | -29.50128 | 1.703781 |

*The cointegration equations in the 3-lag-VEC model*

| Error Correction: | D(LOG(DEFLATOR)) | D(LOG(INVESTMENT)) | D(LOG(M_ONE)) | D(LOG(REALGNP)) | D(LOG(UNEMPL)) |
|---|---|---|---|---|---|
| CointEq1 | 0.012441<br>(0.00656)<br>(1.89511) | -0.074175<br>(0.02735)<br>(-2.71165) | 0.022298<br>(0.00871)<br>(2.56138) | -0.039480<br>(0.01198)<br>(-3.29632) | 0.180855<br>(0.07367)<br>(2.45480) |
| CointEq2 | 0.002520<br>(0.00766)<br>(0.32881) | -0.172149<br>(0.03193)<br>(-5.39081) | 0.002547<br>(0.01016)<br>(0.25066) | -0.037476<br>(0.01398)<br>(-2.68022) | 0.320545<br>(0.08601)<br>(3.72691) |

*The cointegration equations in the 6-lag-VEC model*

| Error Correction: | D(LOG(DEFLATOR)) | D(LOG(INVESTMENT)) | D(LOG(M_ONE)) | D(LOG(REALGNP)) | D(LOG(UNEMPL)) |
|---|---|---|---|---|---|
| CointEq1 | 0.013730<br>(0.00766)<br>(1.79236) | 0.037146<br>(0.03537)<br>(1.05023) | 0.035847<br>(0.01090)<br>(3.28786) | -0.034119<br>(0.01508)<br>(-2.26253) | -0.016933<br>(0.09489)<br>(-0.17845) |
| CointEq2 | 0.056009<br>(0.01103)<br>(5.07877) | -0.158887<br>(0.05092)<br>(-3.12032) | 0.002542<br>(0.01570)<br>(0.16194) | -0.028966<br>(0.02171)<br>(-1.33419) | 0.150096<br>(0.13661)<br>(1.09869) |

***Figure 6: Cointegration relationships of two VECs***

least one cointegration equation. It seems that the trend variable is significant although it has only a slight impact on the outcome (eliminating it worsens the results only slightly) – and at least one cointegration equation is justified. In addition the cointegration relationships provide an opportunity of economic interpretation: If one looks, e.g., at the 6-lag VEC, what effect does the level of GNP have on GNP growth? First of all, the value of Z1 (sum of the components in CointEq1) has more impact on GNP growth ( D(LOG(GNP)) ) than Z2 (-0.034 vs. –0.029). And Z1 is much more (positively) influenced by the log-level of GNP than Z2 is (negatively) dependent on this level. Thus a high GNP level yields a high Z1 and this in return is multiplied with a negative number – so GNP growth is negatively dependent on GNP levels, a productivity slowdown, as mentioned before, may be identified. Monetary expansions ( D(LOG(M_ONE)) ) depend positively on the GNP level – this could be attributed to an ever-increasing usage of this policy tool throughout the sixties, seventies and part of the eighties. A simple regression and a scatter plot confirm this suspicion.

Dependent Variable: DLOG(HM_ONE)
Method: Least Squares

| Variable | Coefficient | Std. Error | t-Statistic | Prob. |
|---|---|---|---|---|
| C | -0.081705 | 0.023684 | -3.449813 | 0.0007 |
| LOG(HREALGNP) | 0.012093 | 0.003055 | 3.958584 | 0.0001 |
| AR(1) | 0.528659 | 0.065810 | 8.033050 | 0.0000 |

| | | | |
|---|---|---|---|
| R-squared | 0.463780 | Mean dependent var | 0.011827 |
| Adjusted R-squared | 0.457358 | S.D. dependent var | 0.009602 |
| S.E. of regression | 0.007073 | Akaike info criterion | -7.047588 |
| Sum squared resid | 0.008354 | Schwarz criterion | -6.992251 |
| Log likelihood | 602.0450 | F-statistic | 72.21971 |
| Durbin-Watson stat | 2.056379 | Prob(F-statistic) | 0.000000 |

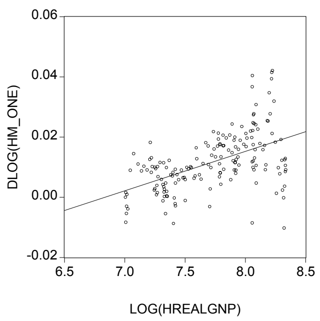


***Figure 7: Money growth and GNP level – regression and scatter plot***

Although the AIC and SC compared to the log-VAR do not show any improvement, the mean error of the dependent variables has decreased. The Theil U table shows that the two models beat the log-VAR in out-of-sample forecasts, especially in the short term and in deflator predictions, whereas the log-VAR performs still remarkably well in long-term forecasts for GNP (Even if the log-VAR is better in predicting unemployment than the VECs, its Theil U values higher than one do not make it a better model than the naïve forecast). By this table it can also be judged that the 6-lag-VEC does not outperform the 3-lag VEC. Although the 6-lag VAR certainly provides more information based on “real values”, more lags add apparently more perturbation than information.

*Estimation output of the 3-lag VEC*

| | Deflator | Investment | M1 | GNP | Unemployment |
|---|---|---|---|---|---|
| R-squared | 0.584126 | 0.506019 | 0.541095 | 0.355857 | 0.561109 |
| Adj. R-squared | 0.534338 | 0.446881 | 0.486156 | 0.278742 | 0.508565 |
| Sum sq. resides | 0.003773 | 0.065514 | 0.006635 | 0.012560 | 0.475237 |
| S.E. equation | 0.005155 | 0.021479 | 0.006836 | 0.009405 | 0.057851 |
| F-statistic | 11.73231 | 8.556500 | 9.848956 | 4.614589 | 10.67897 |
| Log likelihood | 625.3688 | 397.0235 | 580.2101 | 529.1632 | 238.4992 |
| Akaike AIC | -7.592110 | -4.737793 | -7.027626 | -6.389540 | -2.756239 |
| Schwarz SC | -7.246153 | -4.391836 | -6.681669 | -6.043583 | -2.410282 |
| Mean dependent | 0.010286 | 0.007849 | 0.012293 | 0.008017 | 0.002030 |
| S.D. dependent | 0.007554 | 0.028881 | 0.009536 | 0.011074 | 0.082524 |

| | |
|---|---|
| Determinant Residual Covariance | 3.35E-20 |
| Log Likelihood | 2452.183 |
| Akaike Information Criteria | -29.37729 |
| Schwarz Criteria | -27.41686 |

*Estimation output of the 6-lag VEC*

| | Deflator | Investment | M1 | GNP | Unemployment |
|---|---|---|---|---|---|
| R-squared | 0.683685 | 0.556499 | 0.616556 | 0.465232 | 0.588914 |
| Adj. R-squared | 0.602055 | 0.442047 | 0.517603 | 0.327227 | 0.482828 |
| Sum sq. resids | 0.002609 | 0.055627 | 0.005286 | 0.010112 | 0.400401 |
| S.E. equation | 0.004587 | 0.021180 | 0.006529 | 0.009030 | 0.056825 |
| F-statistic | 8.375432 | 4.862292 | 6.230778 | 3.371132 | 5.551260 |
| Log likelihood | 641.1150 | 400.9356 | 585.6969 | 534.7722 | 245.9922 |
| Akaike AIC | -7.746689 | -4.687077 | -7.040725 | -6.392003 | -2.713276 |
| Schwarz SC | -7.104293 | -4.044681 | -6.398330 | -5.749607 | -2.070881 |
| Mean dependent | 0.010590 | 0.008680 | 0.012571 | 0.008234 | -0.001493 |
| S.D. dependent | 0.007272 | 0.028355 | 0.009400 | 0.011010 | 0.079017 |

| | |
|---|---|
| Determinant Residual Covariance | 1.19E-20 |
| Log Likelihood | 2487.221 |
| Akaike Information Criteria | -29.42957 |
| Schwarz Criteria | -25.98399 |

***Figure 8: Estimation outputs of two VECs***

## Seasonality – A VEC in the Differences

In estimating the appropriateness of the VECs, one task remains to be done: a check if the residuals of these models are really white noise (stationary and unrelated). The auto-correlograms for the residuals of the (really) integrated series (here only the GNP series) show no remaining information not captured by the level models. But the correlograms of two series' residuals provide a different picture: the unemployment and the investment series. A quick look at their correlograms (figure 9) shows that a small seasonality component remains. Although it might be unworthy to look for (small) improvements in investment data, the unemployment forecasts seem desperate for any inclusion of seasonality: Their Theil U coefficient never reaches values lower than one for any of the models discussed above.

So, how one could adjust for this additional information in the residuals? The seasonality component could be included by increasing the lag length of the VEC, e.g. eight lags. But this yields even worse results for all of the three variables, due to noisy data.

This leads to leaving the VAR approach and building either auto-regressive models

***VEC (3-lags)***
*Autocorr. of GNP-residuals*

Sample: 1948:1 1990:4
Included observations: 157

| | AC | PAC | Q-Stat | Prob |
|---|---|---|---|---|
| 1 | 0.030 | 0.030 | 0.1436 | 0.705 |
| 2 | 0.044 | 0.043 | 0.4595 | 0.795 |
| 3 | -0.031 | -0.034 | 0.6142 | 0.893 |
| 4 | -0.001 | -0.001 | 0.6143 | 0.961 |
| 5 | 0.010 | 0.013 | 0.6304 | 0.987 |
| 6 | 0.000 | -0.002 | 0.6304 | 0.996 |
| 7 | -0.060 | -0.062 | 1.2385 | 0.990 |
| 8 | -0.103 | -0.100 | 3.0292 | 0.933 |
| 9 | 0.049 | 0.061 | 3.4277 | 0.945 |
| 10 | 0.064 | 0.067 | 4.1155 | 0.942 |
| 11 | 0.041 | 0.026 | 4.4066 | 0.956 |
| 12 | -0.122 | -0.130 | 6.9753 | 0.859 |
| 13 | -0.057 | -0.050 | 7.5479 | 0.872 |
| 14 | 0.018 | 0.035 | 7.6068 | 0.909 |
| 15 | 0.001 | -0.014 | 7.6069 | 0.939 |
| 16 | 0.002 | -0.011 | 7.6076 | 0.960 |
| 17 | 0.077 | 0.103 | 8.6783 | 0.950 |

***VEC (6 lags)***
*Autocorr. of GNP-residuals*

Sample: 1948:1 1990:4
Included observations: 157

| | AC | PAC | Q-Stat | Prob |
|---|---|---|---|---|
| 1 | -0.038 | -0.038 | 0.2253 | 0.635 |
| 2 | 0.029 | 0.028 | 0.3621 | 0.834 |
| 3 | 0.006 | 0.008 | 0.3673 | 0.947 |
| 4 | -0.018 | -0.018 | 0.4204 | 0.981 |
| 5 | 0.026 | 0.024 | 0.5321 | 0.991 |
| 6 | 0.015 | 0.018 | 0.5674 | 0.997 |
| 7 | 0.021 | 0.021 | 0.6381 | 0.999 |
| 8 | -0.250 | -0.251 | 11.084 | 0.197 |
| 9 | 0.014 | -0.004 | 11.115 | 0.268 |
| 10 | -0.025 | -0.011 | 11.224 | 0.340 |
| 11 | -0.014 | -0.012 | 11.260 | 0.422 |
| 12 | -0.162 | -0.186 | 15.800 | 0.201 |
| 13 | -0.009 | -0.007 | 15.813 | 0.259 |
| 14 | -0.043 | -0.027 | 16.138 | 0.305 |
| 15 | 0.006 | 0.017 | 16.145 | 0.372 |
| 16 | 0.141 | 0.075 | 19.665 | 0.236 |
| 17 | 0.045 | 0.073 | 20.032 | 0.273 |

*Autocorr. of unemployment-residuals* (VEC (3-lags))

Sample: 1948:1 1990:4
Included observations: 160

| | AC | PAC | Q-Stat | Prob |
|---|---|---|---|---|
| 1 | -0.019 | -0.019 | 0.0591 | 0.808 |
| 2 | -0.051 | -0.052 | 0.4909 | 0.782 |
| 3 | 0.080 | 0.078 | 1.5374 | 0.674 |
| 4 | -0.141 | -0.142 | 4.8477 | 0.303 |
| 5 | -0.083 | -0.080 | 6.0005 | 0.306 |
| 6 | 0.104 | 0.084 | 7.8184 | 0.252 |
| 7 | 0.090 | 0.110 | 9.1761 | 0.240 |
| 8 | -0.175 | -0.181 | 14.396 | 0.072 |
| 9 | 0.045 | 0.014 | 14.746 | 0.098 |
| 10 | -0.018 | -0.022 | 14.803 | 0.139 |
| 11 | -0.008 | 0.068 | 14.814 | 0.191 |
| 12 | -0.204 | -0.284 | 22.100 | 0.036 |
| 13 | 0.008 | -0.016 | 22.112 | 0.054 |
| 14 | -0.085 | -0.096 | 23.382 | 0.054 |
| 15 | -0.079 | -0.001 | 24.508 | 0.057 |
| 16 | 0.075 | -0.067 | 25.520 | 0.061 |
| 17 | 0.044 | 0.047 | 25.868 | 0.077 |

*Autocorr. of unemployment-residuals* (VEC (6 lags))

Sample: 1948:1 1990:4
Included observations: 157

| | AC | PAC | Q-Stat | Prob |
|---|---|---|---|---|
| 1 | -0.038 | -0.038 | 0.2253 | 0.635 |
| 2 | 0.029 | 0.028 | 0.3621 | 0.834 |
| 3 | 0.006 | 0.008 | 0.3673 | 0.947 |
| 4 | -0.018 | -0.018 | 0.4204 | 0.981 |
| 5 | 0.026 | 0.024 | 0.5321 | 0.991 |
| 6 | 0.015 | 0.018 | 0.5674 | 0.997 |
| 7 | 0.021 | 0.021 | 0.6381 | 0.999 |
| 8 | -0.250 | -0.251 | 11.084 | 0.197 |
| 9 | 0.014 | -0.004 | 11.115 | 0.268 |
| 10 | -0.025 | -0.011 | 11.224 | 0.340 |
| 11 | -0.014 | -0.012 | 11.260 | 0.422 |
| 12 | -0.162 | -0.186 | 15.800 | 0.201 |
| 13 | -0.009 | -0.007 | 15.813 | 0.259 |
| 14 | -0.043 | -0.027 | 16.138 | 0.305 |
| 15 | 0.006 | 0.017 | 16.145 | 0.372 |
| 16 | 0.141 | 0.075 | 19.665 | 0.236 |
| 17 | 0.045 | 0.073 | 20.032 | 0.273 |

***Figure 9: Autocorrelations of (somewhat) seasonal residuals of two VEC models***

for the series or a transfer function for the interesting series (at least for the unemployment rate). But a pre-whitened transfer function for the unemployment rate relies only on lag-one variables, thus making a dynamic forecast not very reliable and difficult to handle (this would need a model relying on forecasts of the pre-whitened input series by some other explaining variables), although it provides a Theil U value of 0.50 for one-step-ahead forecasts. An AR(1,2,4,8,12) model for the unemployment rate, by contrast, yields results better than the others does not outperform the naive forecast. And apart from that, leaving the approach of building models by vector auto-regression leads to drifting away from the basic paper.

So the best choice would be to construct a VAR/VEC based on pre-whitened series: One could then estimate forecasts without having to bother for seasonality and re-apply the reverted pre-whitening filter to the outcomes. This challenging approach would be uneasy to handle and would require sophisticated programming skills – which I have to admit, I do not dispose of.

One choice remains for accounting for the seasonality phenomenon without complicated procedures: Estimating a VEC model not for the logs of the five variables, but for their returns. Like a VAR based on the same returns, such a VEC would still capture the most important relationships between the variables – in fact there are more economic models relating GNP growth and M1 growth rather than their total values. This goes especially for the unemployment rate: In most theoretical works, it is related to growth rates. But this effect is already captured by the VEC in logs, so why loosing the level-effects and accounting for "change of growth" (the second derivative)? A rather intuitive explanation would be that the change in the returns of the variables determines the changes in unemployment. Another, more convincing explanation states the following: If the growth rates of variables behave seasonal (as GNP, investment and unemployment rate do), their seasonality could be adjusted for by taking first differences. So a VEC in the returns, which captures the first and the second derivatives of the dynamics, could be appropriate. Let's have a look if this statement is true.

By the AIC the 4-lag VEC in differences can be judged as the best one.[6] And its performance is striking: Theil U values of mostly less than 0.8 for the unemployment rate may certainly not be the best model but outperform clearly all other alternatives – and, as the only one, even the naïve forecast. These numbers are not pure coincidence: the Theil U values for one-step ahead forecasts of this models are around 0.55 (for ten arbitrarily chosen forecast periods throughout the estimation sample). GNP predictions deliver better values too, although less dramatically. Inflation forecasts, in comparison, are not up to the predictions made by VEC models in log-levels.

The interpretation of the cointegration relationships seems somewhat easier, because one is more used to handle its input variables. E.g. a Keynesian effect can

[6] Whether the unemployment input is estimated in levels or in returns does not make any difference in the AIC for the neither for the unemployment rate nor for the whole model. Only the Theil U values for GNP forecasts are a lot worse in model with unemployment in levels.

be interpreted into the relationship between monetary growth and the change of the GNP growth: Higher monetary growth increases the growth of GDP growth by both cointegration equations,[7] although this effect does not look very significant. And the reverse seems even more true: the higher GNP growth, and the lower the inflation, the more monetary expansion is stepped up (and vv.) – an effect maybe reflecting monetary policy.

Included observations: 158 after adjusting endpoints
Standard errors & t-statistics in parentheses

| Cointegrating Eq: | CointEq1 | CointEq2 |
|---|---|---|
| DLOG(DEFL(-1)) | 1.000000 | 0.000000 |
| DLOG(GNP(-1)) | 0.000000 | 1.000000 |
| DLOG(INV(-1)) | 1.025716 | 0.129988 |
| | (0.50983) | (0.15289) |
| | (2.01187) | (0.85019) |
| DLOG(MONE(-1)) | -0.797505 | -0.119462 |
| | (0.65427) | (0.19621) |
| | (-1.21892) | (-0.60885) |
| DLOG(UNEMP(-1)) | -0.027903 | 0.152327 |
| | (0.08528) | (0.02557) |
| | (-0.32721) | (5.95656) |
| @TREND(48:1) | 1.81E-05 | 3.86E-05 |
| | (9.6E-05) | (2.9E-05) |
| | (0.18968) | (1.34482) |
| C | -0.010558 | -0.011169 |

| Error Correction: | D(DLOG(DEFL)) | D(DLOG(GNP)) | D(DLOG(INV)) | D(DLOG(MONE)) | D(DLOG(UNEMP)) |
|---|---|---|---|---|---|
| CointEq1 | 0.005198 | -0.201133 | -0.930400 | -0.102131 | 1.836803 |
| | (0.03101) | (0.05769) | (0.13066) | (0.04069) | (0.34271) |
| | (0.16765) | (-3.48636) | (-7.12058) | (-2.51001) | (5.35966) |
| CointEq2 | 0.240530 | 0.310223 | 1.367599 | 0.401202 | -7.714211 |
| | (0.11740) | (0.21843) | (0.49472) | (0.15406) | (1.29757) |
| | (2.04878) | (1.42023) | (2.76439) | (2.60419) | (-5.94512) |
| R-squared | 0.397923 | 0.483446 | 0.525241 | 0.405882 | 0.447119 |
| Adj. R-squared | 0.299806 | 0.399267 | 0.447873 | 0.309063 | 0.357020 |
| Sum sq. resids | 0.003585 | 0.012409 | 0.063654 | 0.006173 | 0.437891 |
| S.E. equation | 0.005153 | 0.009587 | 0.021714 | 0.006762 | 0.056953 |
| F-statistic | 4.055620 | 5.743070 | 6.788855 | 4.192163 | 4.962531 |
| Log likelihood | 620.6082 | 522.5100 | 393.3421 | 577.6729 | 240.9897 |
| Akaike AIC | -7.564661 | -6.322911 | -4.687875 | -7.021176 | -2.759363 |
| Schwarz SC | -7.118840 | -5.877090 | -4.242054 | -6.575355 | -2.313543 |
| Mean dependent | 0.000126 | 7.96E-05 | 0.000206 | 5.17E-06 | -0.001682 |
| S.D. dependent | 0.006158 | 0.012370 | 0.029223 | 0.008135 | 0.071026 |

| | |
|---|---|
| Determinant Residual Covariance | 3.33E-20 |
| Log Likelihood | 2422.022 |
| Akaike Information Criteria | -29.05091 |
| Schwarz Criteria | -26.58920 |

***Figure 10: Estimation output of a VEC in returns (3 lags, 2 cointegration equations)***

So, with respect to unemployment, the 4-lag VEC in the differences seems to be a better choice than in the levels. Nevertheless, omitting the level effects in order to

[7] The effect of an *increase* in M1 expansion is negligible (its coefficients are very close to zero).

catch more of the growth dynamics did not yield better results for GNP forecasts, and seems to get away from describing the process forming the GNP deflator.

## Conclusion

By comparison of the Theil U statistics of the considered models it can be seen that the models with more lags perform better in the long run. Whereas the VECs improve forecasts of deflator and unemployment rates considerably, the standard log-VAR shows comparable values concerning GNP predictions. Considering Simkins' BVAR, the 3-lag VEC delivers comparable Theil U values, especially in the long run (whereas in the short run the random-walk specification of BVAR certainly strikes more). Only short-term GNP predictions remain a strong feature of the standard BVAR versus the more sophisticated regression models.

Concerning the unemployment rate, the VEC in differences seems the only model producing usable results. This may be attributed to the fact that the unemployment rate is more dependent on the slope and acceleration of the other four variables than on their levels – economic models relate it more with growth ratios than with total indicators, too.

Thus the results of Simkins' restricted VAR would have been much more competitive with respect to the random-walk BVARs, if Prof. Simkins would have chosen a more appropriate specification of his VAR, or even a more sophisticated VEC model (which would have changed the random drawing procedure only by a slight amount).

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EViews Help: see Software

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Simkins S.: "*Do real business cycle models really exhibit business cycle behaviour?*" in: Journal of Monetary Economics 33, p 381-404; 1994

Simkins, S.: "*Forecasting with vector autoregressive (VAR) models subject to business cycle restriction*s", in: International Journal of Forecasting 11/1995 p 569-583; 1995

### *Software:*

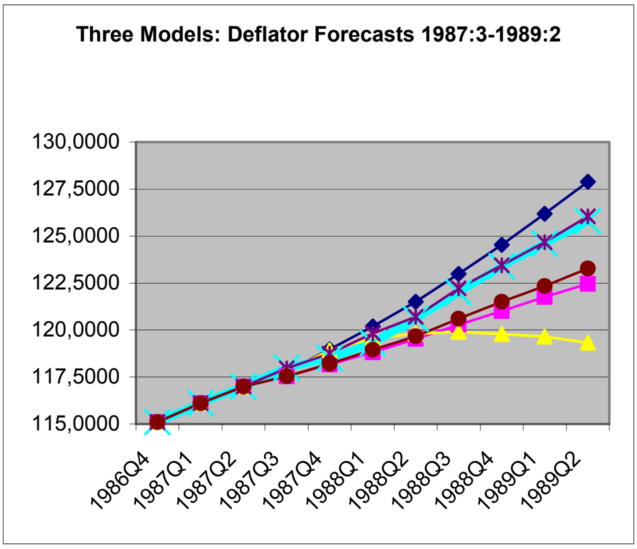
EViews 3.1, Quantitative Micro Software, 1994-1998

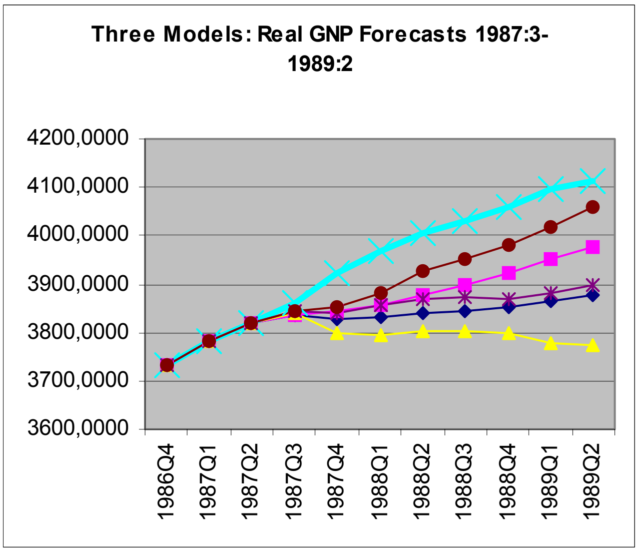
MS Excel 97, Microsoft Corporation 1985-1997

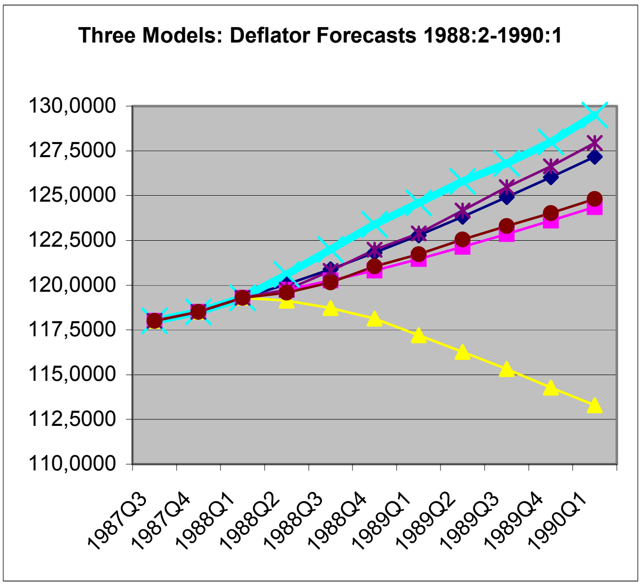
# Appendix 1: Forecasting Results Of the Considered Models

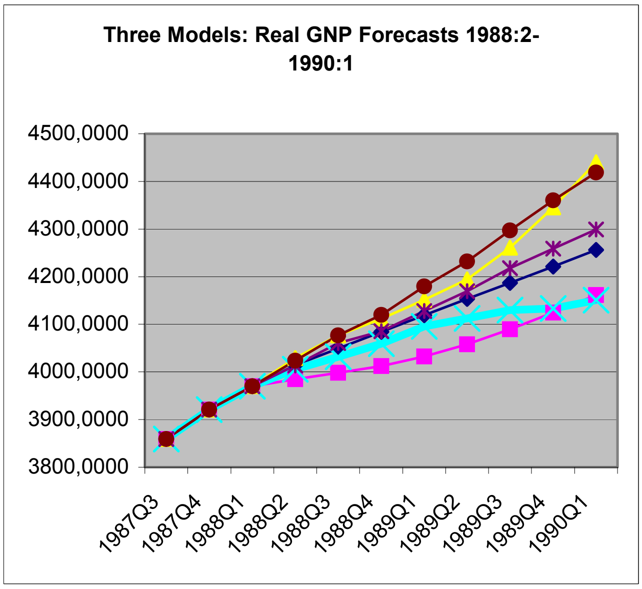
| | Step ahead | level-VAR (6 lags) | log-VAR (3 lags) | VEC (3 lags) | VEC (6 lags) | VEC in returns (4 lags) | AR-model for unemployment rate |
|---|---|---|---|---|---|---|---|
| GNP deflator | 1 | 0.551 | 0.375 | 0.287 | 0.313 | 0.451 | |
| | 2 | 0.640 | 0.389 | 0.279 | 0.274 | 0.456 | |
| | 3 | 0.700 | 0.400 | 0.255 | 0.241 | 0.443 | |
| | 4 | 0.756 | 0.405 | 0.230 | 0.224 | 0.424 | |
| | 5 | 0.806 | 0.404 | 0.211 | 0.212 | 0.407 | |
| | 6 | 0.845 | 0.397 | 0.195 | 0.194 | 0.390 | |
| | 7 | 0.879 | 0.388 | 0.185 | 0.177 | 0.379 | |
| | 8 | 0.913 | 0.382 | 0.184 | 0.167 | 0.373 | |
| Real GNP | 1 | 1.059 | 1.040 | 0.859 | 0.983 | 0.885 | |
| | 2 | 1.191 | 1.032 | 0.877 | 0.957 | 0.862 | |
| | 3 | 1.194 | 1.003 | 0.843 | 0.911 | 0.791 | |
| | 4 | 1.176 | 0.962 | 0.808 | 0.879 | 0.735 | |
| | 5 | 1.155 | 0.914 | 0.780 | 0.865 | 0.709 | |
| | 6 | 1.142 | 0.866 | 0.760 | 0.857 | 0.715 | |
| | 7 | 1.140 | 0.817 | 0.743 | 0.846 | 0.740 | |
| | 8 | 1.143 | 0.775 | 0.734 | 0.836 | 0.781 | |
| Unemployment Rate | 1 | 3.137 | 1.178 | 1.251 | 1.293 | 0.885 | 0.940 |
| | 2 | 4.013 | 1.539 | 1.716 | 1.663 | 0.862 | 1.098 |
| | 3 | 4.952 | 1.877 | 2.157 | 2.191 | 0.791 | 1.267 |
| | 4 | 5.740 | 2.013 | 2.417 | 2.627 | 0.735 | 1.426 |
| | 5 | 6.539 | 2.104 | 2.647 | 3.031 | 0.709 | 1.567 |
| | 6 | 7.077 | 2.079 | 2.778 | 3.223 | 0.715 | 1.666 |
| | 7 | 7.467 | 1.993 | 2.848 | 3.316 | 0.740 | 1.733 |
| | 8 | 7.693 | 1.883 | 2.873 | 3.332 | 0.781 | 1.761 |

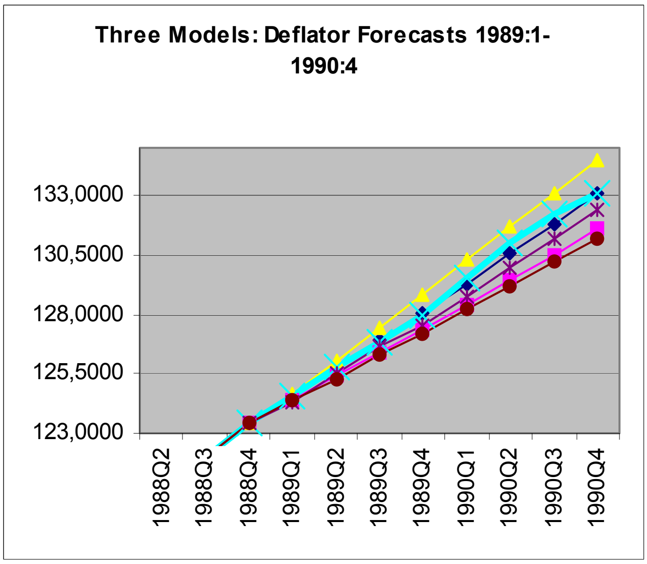
# Appendix 2: Model Forecasts for Three Periods in Diagrams

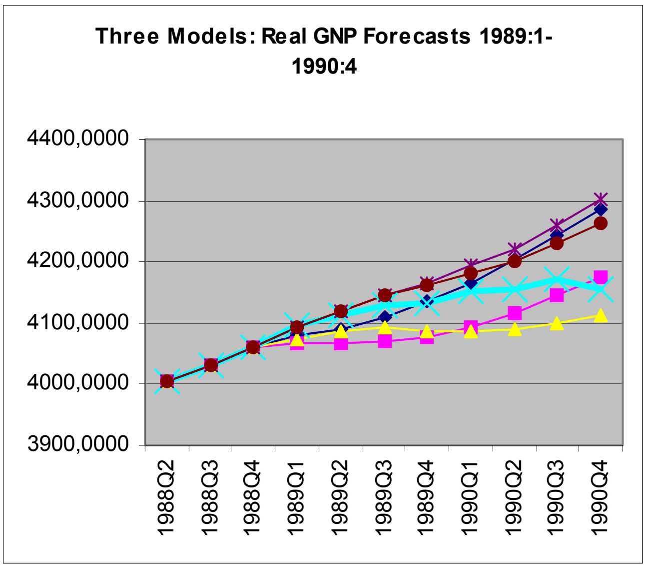


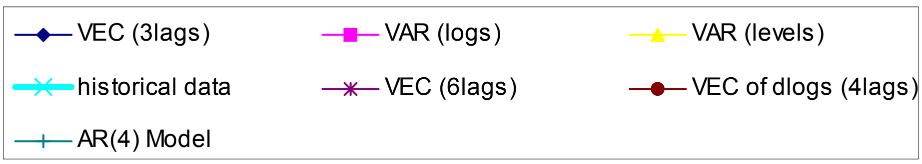



## Appendix 2: Model Forecasts for Three Periods in Diagrams (continued)